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TRANSVERSE VIBRATIONS OF AN ANNULAR CIRCULAR PLATE WITH FREE EDGES AND AN INTERMEDIATE CONCENTRIC CIRCULAR SUPPORT

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1. INTRODUCTION

The fundamental frequency of transverse vibration of a solid circular plate with an intermediate circular support and free edge was determined by Bodine in a well known paper [1]. Recently performed calculations [2] have shown that the fundamental frequency coefficients determined in reference [1] do not possess sufficient accuracy from the point of view of modern design needs.

The present study deals with the determination of the exact fundamental eigenvalue of transverse vibration of the structural system shown in Figure 1. In order to assess the validity and accuracy of the frequency coefficients* an independent approximate solution was also obtained by means of a variational formulation whereby the displacement amplitude was expressed in terms of polynomial co-ordinate functions which satisfy the essential boundary condition and null bending moment at the outer edges. It is important to point out that the present problem has not, apparently, been considered by other researchers [3].

2. EXACT SOLUTION

In the case of normal, axisymmetric modes of transverse vibration the amplitudes are expressed in terms of Bessel functions [3] (see Figure 1)

$$W_1 = A_1\mathrm{J}_0(kr) + B_1\mathrm{Y}_0(kr) + C_1\mathrm{I}_0(kr) + D_1\mathrm{K}_0(kr), \quad b \leqslant r \leqslant c, \tag{1}$$

$$W_2 = A_2\mathrm{J}_0(kr) + B_2\mathrm{Y}_0(kr) + C_2\mathrm{I}_0(kr) + D_2\mathrm{K}_0(kr), \quad c \leqslant r \leqslant a, \tag{2}$$

where $k = \sqrt[4]{(\rho h/D)}\sqrt{\omega}$.

*It was felt that this was a convenient decision in view of the complexity of the exact analytical solution which, as it will be seen, requires solving an (8×8) determinantal equation in terms of ordinary and modified Bessel functions of zero order and their derivatives.

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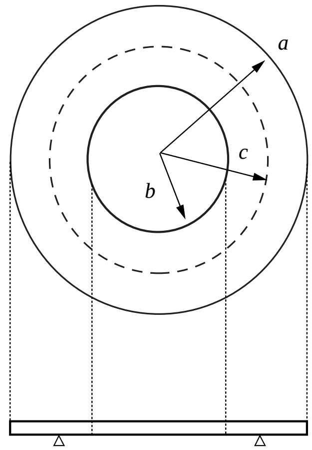

Figure 1. Circular annular plate executing transverse, axisymmetric vibrations considered in the present study.

The determinantal equation is generated setting up the appropriate boundary and continuity conditions:

$$(\mathrm{d}^2 W_1/\mathrm{d}r^2 + (\nu/r)\mathrm{d}W_1/\mathrm{d}r)|_{r=b} = 0, \tag{3a}$$

$$(\mathrm{d}/\mathrm{d}r)\nabla^2 W_1|_{r=b} = 0, \quad W_1(c) = 0, \quad W_2(c) = 0, \tag{3b–d}$$

$$\mathrm{d}W_1/\mathrm{d}r(c) = (\mathrm{d}W_2/\mathrm{d}r)(c), \quad (\mathrm{d}^2 W_1/\mathrm{d}r^2)(c) = (\mathrm{d}^2 W_2/\mathrm{d}r^2)(c), \tag{3e, f}$$

$$(\mathrm{d}^2 W_2/\mathrm{d}r^2 + (\nu/r)\mathrm{d}W_2/\mathrm{d}r)_{r=a} = 0, \quad \mathrm{d}/\mathrm{d}r\nabla^2 W_2|_{r=a} = 0. \tag{3g, h}$$

Setting up the frequency equation and then obtaining the fundamental frequency coefficient $(k_1 a)^2 = \sqrt{(\rho h/D)}\omega_1 a^2$ has been greatly facilitated by the use of MAPLE [4].

3. APPROXIMATE ANALYTICAL SOLUTION

In order to obtain an independent analytical solution the optimized Rayleigh–Ritz method was employed to determine upper bounds of the exact fundamental eigenvalue.

The displacement amplitude has been approximated by means of

$$W \cong W_a = \sum_{j=1}^{4} C_j \varphi_j(r), \tag{4}$$

where $\varphi_j = \alpha_j r^{p+j-1} + \beta_j r^{j+2} + \gamma_j r^{j+1} + 1$ and p is Rayleigh's optimization parameter [5]. The α's, β's and γ's are determined substituting each co-ordinate function in equations (3a), (3c) and (3g).

TABLE 1

Fundamental frequency coefficients of the system shown in Figure 1. *Note*: ( ), *Rayleigh–Ritz polynomial approach*

	c/a							
b/a	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
0·1	3·9790 (5·17)	4·6188 (5·46)	5·4983 (6·12)	6·7206 (7·35)	8·1687 (9·10)	8·7380 (10·06)	7·6434 (8·78)	6·1193
0·2	–	4·0647 (4·17)	4·9833 (5·07)	6·2194 (6·28	7·7052 (7·80)	8·4347 (8·57)	7·4559 (7·56)	5·9612
0·3	–	–	4·3366 (4·35)	5·5798 (5·59)	7·1899 (7·19)	8·3399 (8·35)	7·5635 (7·57)	5·9782
0·4	–	–	–	4·8631 (4·87)	6·5760 (6·58)	8·3761 (8·38)	8·1054 (8·11)	6·2901
0·5	–	–	–	–	5·7861 (5·79)	8·2114 (8·21)	9·1699 (9·17)	7·0440
0·6	–	–	–	–	–	7·4639 (7·46)	10·5683 (10·57)	8·5810
0·7	–	–	–	–	–	–	10·9127	11·827
0·8	–	–	–	–	–	–	–	18·444

The generation of the frequency determinant follows the usual Rayleigh–Ritz energy procedure and the final step is the minimization of the fundamental frequency coefficient with respect to p,

$$d\Omega_1/dp = 0. \tag{5}$$

4. NUMERICAL RESULTS

All the numerical determinations have been performed for Poisson's ratio (ν) equal to 0·3. Table 1 depicts the fundamental frequency coefficients Ω_1 for different values of the parameters b/a and c/a obtained by the exact approach. In several instances an upper bound obtained by means of the optimized Rayleigh–Ritz method is also shown in the Table. It is observed that for $b/a = 0{\cdot}1$ the upper bounds are rather high but for larger values of b/a, say $b/a = 0{\cdot}3$, the agreement with the exact eigenvalues is extremely good.

It is important to emphasize the fact that for each value of b/a, the maximum value of Ω_1 corresponds to a certain c/a which is the nodal line of the first axisymmetric mode of the completely free circular annular plate. This was verified in several cases, the results being in excellent agreement with eigenvalues available in the open literature [3].

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